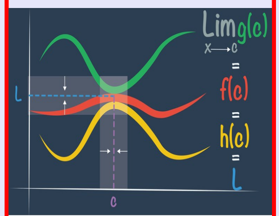
Calculus I

Lecture 31



Feb 19-8:47 AM

If $x^2+y^2+z^2=9$, $\frac{dx}{dt}=5$, and $\frac{dy}{dt}=-4$, find $\frac{dz}{dt}$ when $(x,y,z)=(2,2,1)$.

$$\frac{d}{dt}\left[x^2+y^2+z^2\right]=\frac{d}{dt}[9]$$

$$2x\cdot\frac{dx}{dt}+2y\frac{dy}{dt}+2z\frac{dz}{dt}=0$$

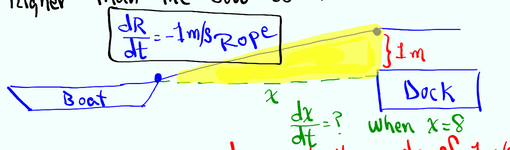
$$2\cdot5+2\cdot(-4)+1\frac{dz}{dt}=0$$

$$\frac{dz}{dt}=-10+8$$

$$\boxed{\frac{dz}{dt}=-2}$$

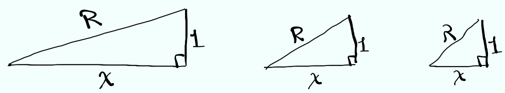
Oct 23-7:23 AM

A boat is pulled into a dock by a rope attached to bow of the boat passing through a pulley on the dock 1 m higher than the bow of the boat.



The rope is pulled in at the rate of 1 m/s.

How fast is the boat approaching the dock when $\boxed{\text{it is 8 m from the dock}}$?



$x^2 + 1^2 = R^2$

$8^2 + 1^2 = R^2$

$2x\frac{dx}{dt} + 0 = 2R\frac{dR}{dt}$

$65 = R^2 \quad R = \sqrt{65}$

$8 \cdot \frac{dx}{dt} = \sqrt{65} \cdot (-1)$

$\frac{dx}{dt} = -\frac{\sqrt{65}}{8} \approx -1$

Oct 23-7:34 AM

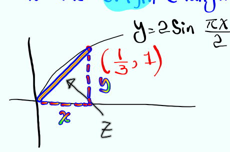
An object is moving along the curve $y = 2\sin\frac{\pi x}{2}$.

As it passes through the point $\boxed{(\frac{1}{3}, 1)}$

$\boxed{x \text{ increases at the rate of } \sqrt{10} \text{ cm/s.}}$

How fast is the distance from the object to the origin changing?

$\frac{dx}{dt} = \sqrt{10}$



$x^2 + y^2 = z^2$

$x^2 + \left[2\sin\frac{\pi x}{2}\right]^2 = z^2$

$x^2 + 4\sin^2\frac{\pi x}{2} = z^2$

$2x\frac{dx}{dt} + 4 \cdot 2\sin\frac{\pi x}{2} \cdot \cos\frac{\pi x}{2} \cdot \frac{\pi}{2} \cdot \frac{dx}{dt} = 2z\frac{dz}{dt}$

$x\frac{dx}{dt} + \pi \cdot 2\sin\frac{\pi x}{2}\cos\frac{\pi x}{2} \cdot \frac{dx}{dt} = z\boxed{\frac{dz}{dt}}$

$x^2 + 4\sin^2\frac{\pi x}{2} = z^2$

$\left(\frac{1}{3}\right)^2 + 4\sin^2\frac{\pi}{6} = z^2$

$\frac{1}{9} + \cancel{4} \cdot \frac{1}{\cancel{4}} = z^2$

$z^2 = \frac{10}{9}$

$z = \frac{\sqrt{10}}{3}$

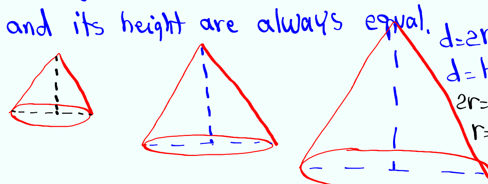
$\frac{1}{3} \cdot \sqrt{10} + \pi \cdot 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{10} = \frac{\sqrt{10}}{3}\frac{dz}{dt}$

$\frac{dz}{dt} = ?$

Oct 23-7:44 AM

$\frac{dV}{dt} = 30\,ft^3/min$

Gravel is being dumped from a belt at the rate of $30\ ft^3/min.$ and it is forming a cone whose base diameter and its height are always equal.



$d = 2r$

$d = h$

$2r = h$

$r = \frac{h}{2}$

How fast is the height increasing when the pile is 10 ft high?

Volume for Cone $\quad V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h \qquad V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$

$30 = \frac{\cancel{3}\pi}{\cancel{12}_{4}} \cdot \cancel{10^2}^{25}\,\frac{dh}{dt}$

Solve for

$\frac{dh}{dt} = \frac{30}{25\pi}$

$= \frac{6}{5\pi}\ ft/min.$

Oct 23-8:01 AM

Two sides of a triangle are 4m & 5m in length and the angle between them is increasing at .06 Rad/s.

$\frac{d\theta}{dt} = .06\ Rad/s$

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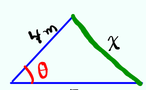


Law of Consines

At what rate the third side is changing when $\boxed{\theta \text{ is } \frac{\pi}{3}}$?

$x^2 = 4^2 + 5^2 - 2\cdot4\cdot5\cdot\cos\theta$

$x^2 = 16 + 25 - 40\cos\frac{\pi}{3}$

$= 41 - 40\cdot\frac{1}{2}$

$2x\frac{dx}{dt} = 0 + 0 - 40\cdot(-\sin\theta)\cdot\frac{d\theta}{dt}$

$x^2 = 21$

$x = \sqrt{21}$

$x\frac{dx}{dt} = 20\sin\theta\frac{d\theta}{dt}$

$\sqrt{21}\frac{dx}{dt} = 20\cdot\sin\frac{\pi}{3}\cdot(.06)$

$\boxed{\frac{dx}{dt} = ?}$

$\sqrt{21}\frac{dx}{dt} = \cancel{20}^{10}\cdot\frac{\sqrt{3}}{\cancel{2}}\cdot(.06)$

$\sqrt{21}\frac{dx}{dt} = .6\sqrt{3}$

$\frac{dx}{dt} = \frac{.6\sqrt{3}}{\sqrt{3}\sqrt{7}}$

$= \frac{6}{10}\cdot\frac{\sqrt{7}}{7}$

$= \frac{3\sqrt{7}}{35}$

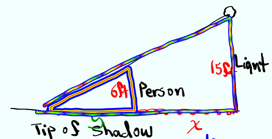
Oct 23-8:13 AM

A street light is 15-ft tall.

A person 6-ft tall walks away from the light at speed of 5 ft/sec. along a Straight Path.

How fast is the tip of person's shadow changing when the person is 40 ft from the light.

$$\frac{y}{\cancel{6}\,2} = \frac{x+y}{\cancel{15}\,5}$$



$\frac{dx}{dt} = 5$

$5y = 2x + 2y$

$3y = 2x$

Now take derivative with respect to time.

$3\frac{dy}{dt} = 2\frac{dx}{dt}$ $\quad 3 \cdot \frac{dy}{dt} = 2 \cdot 5$ $\quad \frac{dy}{dt} = \frac{10}{3}$ ft/s

y is the length of the Shadow.

Tip of shadow

$$\frac{d}{dt}(x+y) = \frac{dx}{dt} + \frac{dy}{dt} = 5 + \frac{10}{3} = \boxed{\frac{25}{3} \text{ ft/s}}$$

Oct 22-8:07 AM